

Module 1D

Solving for Angles & Dihedral Angles

This learning module will explore the basic methods used in the solution of angles and dihedral angles (angles between two planes or flat surfaces). Since the previous modules (*Module 1A*, *1B* and *1C*) have explored in great detail the techniques of using AutoCAD tools in two-dimensional projection, we will focus on the basic strategy of solving this type of descriptive geometry problems in two-dimensional projection, rather than on detailed AutoCAD skills, in this module. Therefore, students who are not comfortable with these basic AutoCAD techniques should review *Module 1A*, *1B* and *1C* first before proceeding to this Module.

Section 1

Determining the Angle between a Line and a Plane (Given: the Line and the Plane in Two Principal Views, Both Showing the Line and the Plane as Foreshortened)

Determining the angle between a line and a plane through several auxiliary views

In this Section, two adjacent views of an oblique plane 1-2-3 and an oblique line A-B are given (*Figure 1D-1A*). The key to determining the true angle between the plane and the line is to draw several auxiliary views so as to obtain the edge view of the plane and the true-length view of the line. Once these views are available, both plane and line appear as angled lines in a two-dimensional drawing view, which allows the true angle between the plane and the line to be measured.

To start, in the *Points, Lines, & Planes in the Three-Dimensional Space.dwg* file (inside the *Given Problems* folder, in the *Student Descriptive Geometry Learning Resource CD*); use the **Zoom** tool to zoom in the area under the tile of *Determining the Angle between a Line and a Plane (Given: the Line and the Plane in Two Principal Views, Both Showing the Line and the Plane as Foreshortened)*; and keep the **ORTHO** and **OTRACK** buttons beneath the **Command Line** turned on, and the **Node** running **Object Snap** in the **Drafting Settings** window turned on. The step-by-step procedures are explained as follows:

Step 1: Creating the Primary Auxiliary view (the edge view of the plane 1-2-3):

First, create the Primary Auxiliary view to obtain the edge view of the plane 1-2-3 (*Figure 1D-1B*). Since a true-length line is needed to project the primary Auxiliary view

from, but none of the edges of the plane 1-2-3 is shown at true-length in the given views, a true-length line must be created first on the plane 1-2-3. To facilitate the solution of this problem, first use a variety of AutoCAD tools (such as the **Line** tool) and settings (such as **ORTHO**) to create the horizontal construction line 1-x parallel to the H/F folding line in the Top view; and from the point x, draw projection line downward into the Front view, which intersects the edge line 2-3 at intersecting point x; then create the true-length (TL) line 1-x on the plane 1-2-3 in the Front view. Save the file as *Angle between a Line and a Plane* in an appropriate folder. Save often throughout this Module. Save the file.

Next, align the **UCS** with TL line 1-x in the Front view and turn **ORTHO** on; draw projection lines from points A, B, x (1), 2 and 3 into the Primary Auxiliary view; draw H/1 Folding Line (Primary Auxiliary View Ref Line) as shown. The first set of dimensions is projected from the Front view for the Primary Auxiliary view.

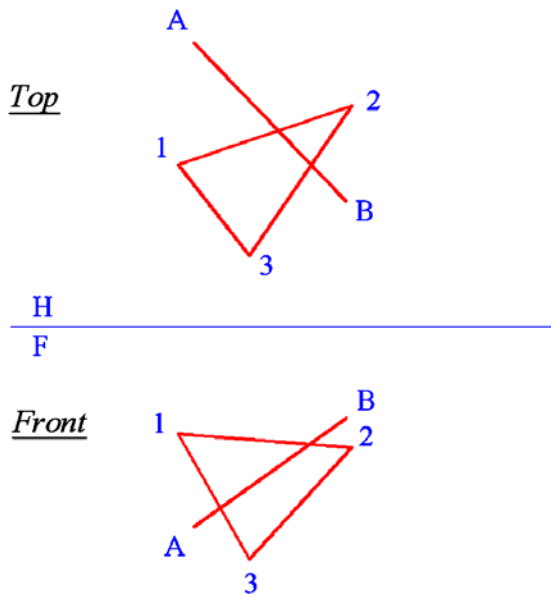


Figure 1D-1A: The given two adjacent views of an oblique plane 1-2-3 and an oblique line A-B.

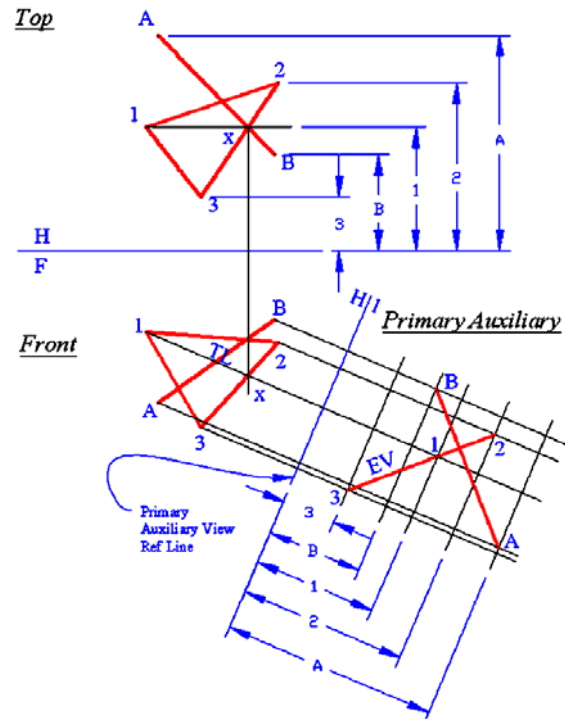


Figure 1D-1B: Creating the Primary Auxiliary view (the edge view of the plane 1-2-3).

Next, obtain the second set of dimensions for the Primary Auxiliary view, from the Top view (the view adjacent to the Front view, from where the first set of dimensions is projected). Return the **UCS** to the **World** settings; use **Linear Dimension** tool to measure, in the Top view, the distances between the H/F Folding Line and points 1, 2, 3,

A and B respectively; use the **Offset** tool to set off the same distances from the H/1 Folding Line (Primary Auxiliary View Ref Plane), in the Primary Auxiliary view. Save the file.

Next, cross-referencing among the Top, Front and Primary Auxiliary views, determine the related points and draw line A-B and plane 1-2-3 in the Primary Auxiliary view. Plane 1-2-3 is now in edge view (abbreviated as EV).

Step 2: Creating the Secondary Auxiliary view (the true shape view or TL of the plane 1-2-3):

Next, create the Secondary Auxiliary view to obtain the true shape view of the plane 1-2-3 (*Figure 1D-1C*). Use the **UCS** tool with the **3-Point** option to align the **UCS** with the EV (edge view line) of plane 1-2-3 in the Primary Auxiliary view and turn **ORTHO** on; then use the **Line** tool and **Endpoint Object Snap** to create projection lines from points 1, 2, 3, A and B into the Secondary Auxiliary view, and the 1/2 Folding Line (Secondary Auxiliary View Ref Plane), which represents the intersection between the viewing planes of Primary and Secondary Auxiliary views. The first set of dimensions is projected from the Primary Auxiliary view for the Secondary Auxiliary view.

Next, obtain the second set of dimensions for the Secondary Auxiliary view, from the Front view (the view adjacent to the Primary Auxiliary view, from where the first set of dimensions is projected). Align the **UCS** with line segment 1-x in the Front view; use **Linear Dimension** tool to measure the distances from H/1 Folding Line (Primary Auxiliary Ref Plane) to points 1, 2, 3, A, and B respectively; then use the **Offset** tool to set off the same distances from the 1/2 Folding Line respectively, in the Secondary Auxiliary view. Next, cross-referencing among the Front, Primary and Secondary Auxiliary views, determine the related points in the Secondary Auxiliary view; use the **Line** tool to draw line A-B and plane 1-2-3 in the Secondary Auxiliary view. Plane 1-2-3 is now in true-shape view (abbreviated as TS).

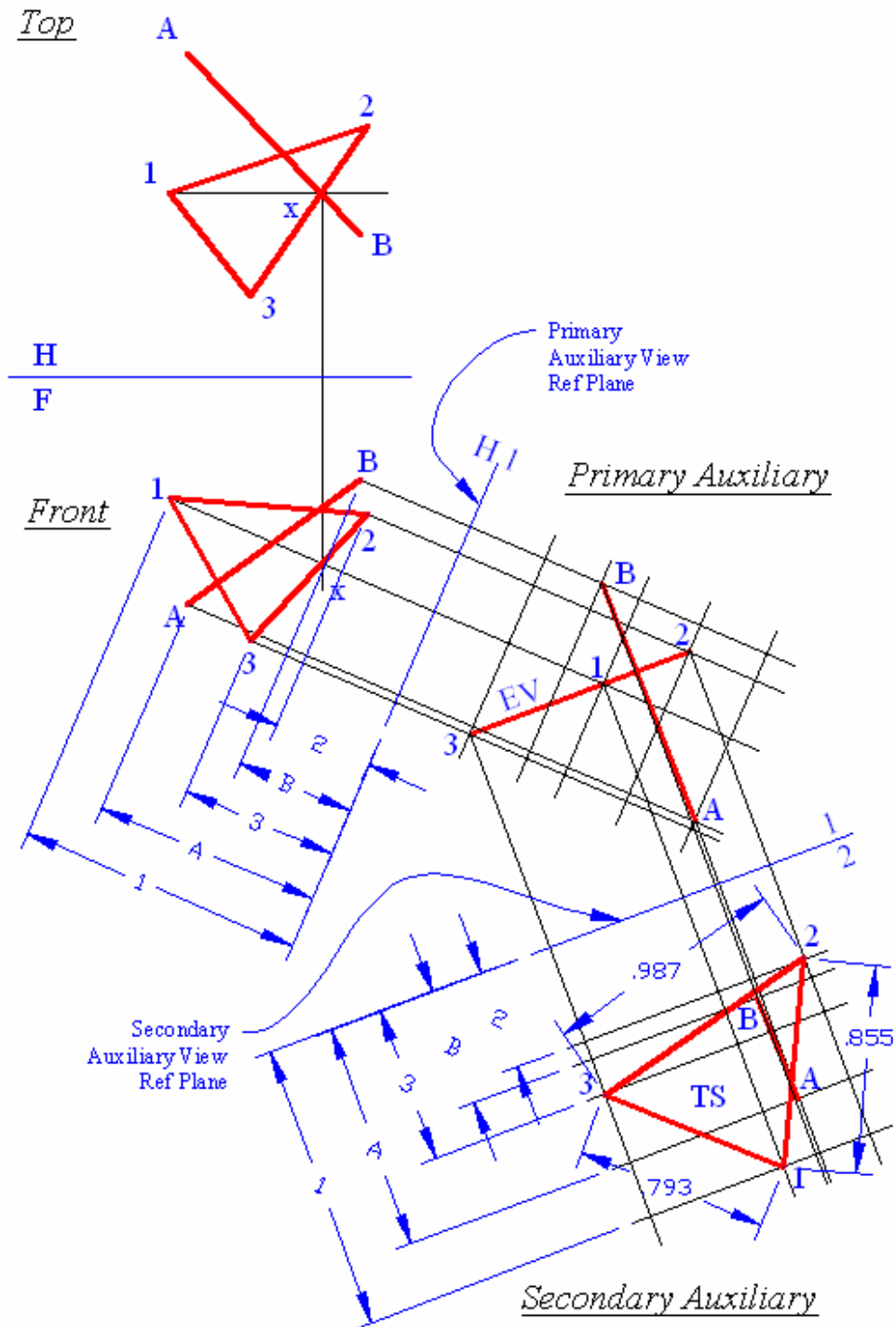


Figure 1D-1C: Creating the Secondary Auxiliary view to obtain the true shape view of the plane 1-2-3.

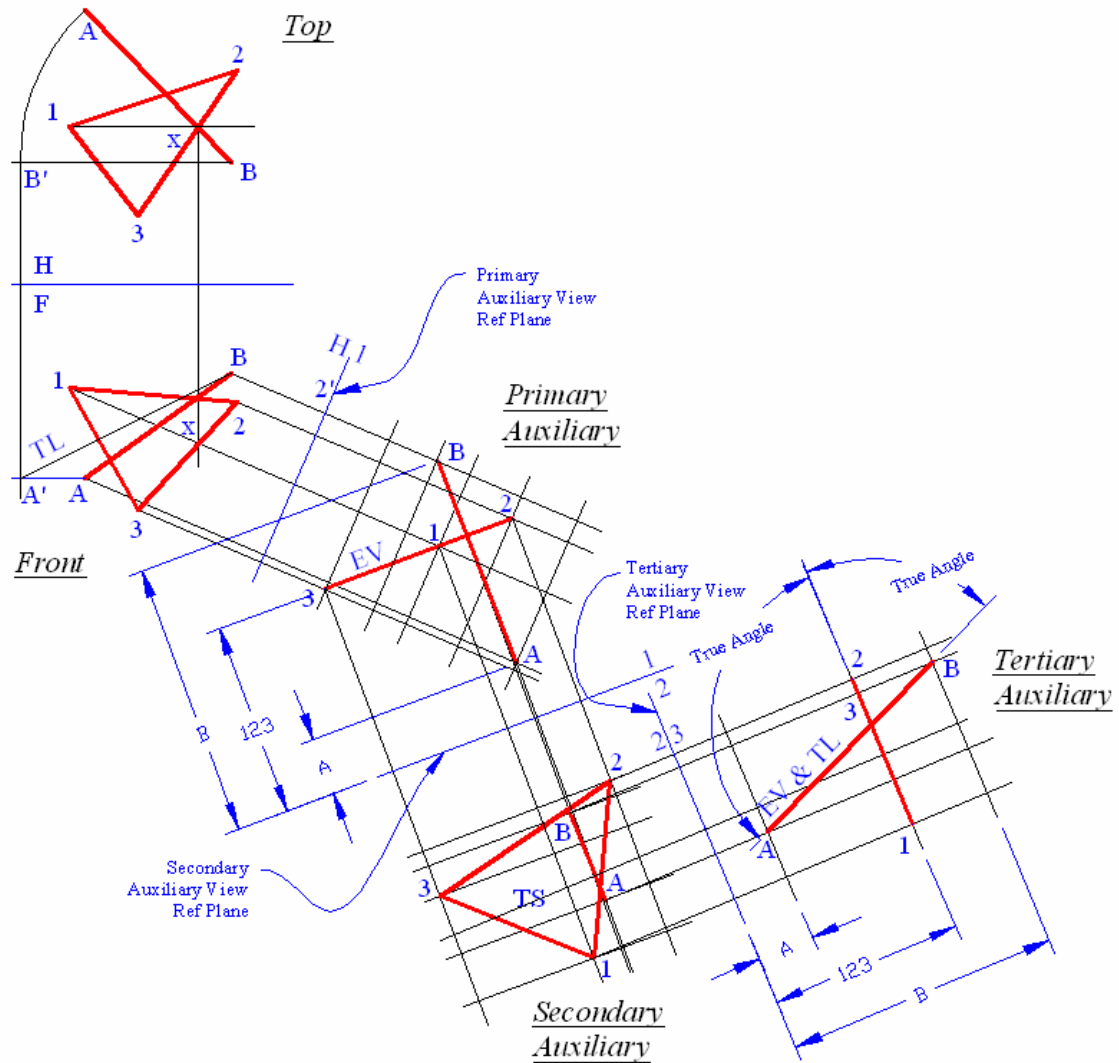


Figure 1D-1D: Creating the Tertiary Auxiliary view to obtain the true-length view of line A-B and an edge view of the plane 1-2-3, and the true angle between the line A-B and the plane 1-2-3.

Step 3: Creating the Tertiary Auxiliary view (the edge view or EV of the plane 1-2-3):

The next step is to create the Tertiary Auxiliary view parallel to line A-B in Secondary Auxiliary view to obtain the true-length view of line A-B and an edge view of the plane 1-2-3 (with foreshortened edges that coincide into a line), and the true angle between the line A-B and the plane 1-2-3, as shown in *Figure 1D-1D*.

First, use the **UCS** tool with the **3-Point** option and **Endpoint Object Snap** to align the **UCS** with the endpoints of line A-B in the Secondary Auxiliary view and turn **ORTHO** on; then use the **Line** tool to create projection lines from points 1, 2, 3, A and B into the Tertiary Auxiliary view, and the 2/3 Folding Line (Tertiary Auxiliary View Ref Plane) perpendicular to these projection lines as shown in *Figure 1D-1D* (the middle portion of the Figure). The first set of dimensions is projected from the Secondary Auxiliary view for the Tertiary Auxiliary view.

Next, obtain the second set of dimensions for the Tertiary Auxiliary view, from the Primary Auxiliary view (the view adjacent to the Secondary Auxiliary view, from where the first set of dimensions is projected). Use the **UCS** tool with the **3-Point** option and **Endpoint Object Snap** to align the **UCS** with the 1/2 Folding Line (Secondary Auxiliary View Ref Plane). Next, with **ORTHO** on, use the **Linear Dimension** tool to measure the distances from 1/2 Folding Line to points 1, 2, 3, A, and B in the Primary Auxiliary view respectively; then use the **Offset** tool to set off the same distances from the 2/3 Folding Line (Tertiary Auxiliary View Ref Plane) respectively. Next, cross-referencing among the Primary, Secondary and Tertiary Auxiliary views, determine the related points in the Tertiary Auxiliary view; then use the **Line** tool to create line A-B and plane 1-2-3 in the Tertiary Auxiliary view. Now line A-B is in a true-length edge view (abbreviated as EV & TL as shown), and plane 1-2-3 is in edge view although it is foreshortened in *Figure 1D-1D* (the lower-right corner).

Last, use the **Angular Dimension** tool to measure the true angles between line A-B and plane 1-2-3.

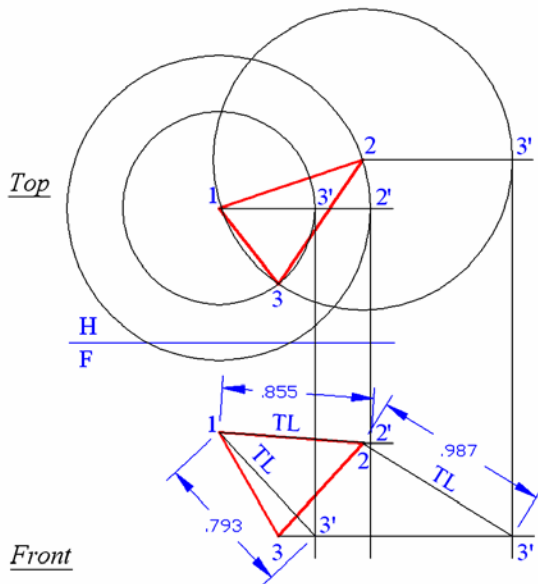


Figure 1D-1E: finding the true-length of edges 1-2 and 1-3 of a plane by the method of revolution.

Review: finding the true-length of edges of a plane by revolution

The method of finding a true-length line 1-x on the plane 1-2-3 in the Front view, as explored in Step 1: Creating the Primary Auxiliary view (the edge view of the plane 1-2-3) in this Section is based on the method of finding the true-length line by revolution, explained as follows:

The true lengths of the edges 1-2, 2-3, and 1-3 of the plane 1-2-3 can be found by the revolution method. We will use the edge lines 1-3 and 1-2 of the plane 1-2-3 as examples. As shown in *Figure 1D-1E*, a “stationary” point (point 1 in the Top view) is first assumed, which serves as the center for creating construction circles with circumferences passing through “revolved” points (point 2 and 3), which intersect at point 2' and 3' with the horizontal lines drawn from the “revolved” points 2 and 3 (these horizontal lines drawn in the Top view represent reference planes parallel to the Frontal viewing plane. This step graphically revolves the edge lines 1-3 and 1-2 in the Top view to a position that is parallel to the Frontal viewing plane, and therefore, allows the edge lines 1-3 and 1-2 to be shown at true-length in the Front view. Next, vertical projection lines are drawn downward from the “revolved” points 2' and 3' in the Top view into the Front view, which intersect at points 2' and 3' with the horizontal lines drawn from “revolved” points 2 and 3 in the Front view; these horizontal lines represent, in the Front view, the reference planes parallel to the Horizontal (Top) viewing plane. Next, TL edge lines 1-2' and 1-3' are created in the Front view. The true-length edge line 2-3' is drawn in the same manner.

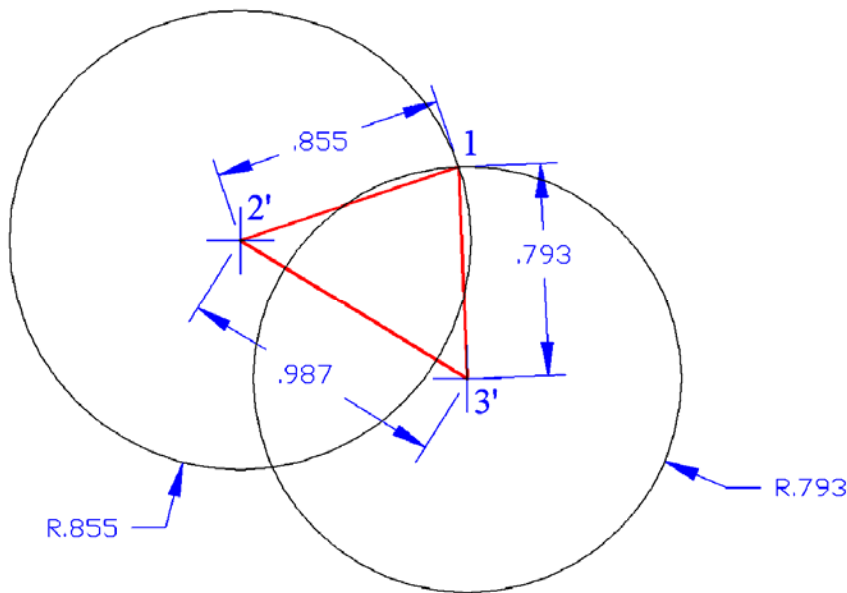


Figure 1D-1F: Completing the true-shape plane with the “triangulation” method.

Once the TL of edge lines 1-2', 1-3' and 2-3' are obtained, the triangular plane 1-2-3 can be created in TS (true-shape) through a geometric construction method called "triangulation." In the "triangulation" method, the TL of one edge (line 2'-3') is copied to a convenient place; then construction circles with radii that equal the TL of the other two edges (edges 1-2' and 1-3') are created, centered at the endpoints 2' and 3' of the first edge line, and intersecting at point 1 as shown. Next, edge lines 1-2' and 1-3' are created to complete the plane 1-2-3 in true-shape, as shown in *Figure 1D-1F*. Save the file.

Section 2

Determining the Angle between Two Planes (The Dihedral Angle) with Intersection between Two Planes Given

In three-dimensional space, there are three possible relationships between two planes (*Figure 1D-2A*):

- **Parallelism**: Both planes are parallel to each other and do not intersect;
- **Perpendicularity**: Both plane actually intersect or have the potential of intersecting each other at a straight angle (a 90° angle) if extended;
- **Angularity**: Both plane actually intersect or have the potential of intersecting each other at an oblique angle (any angle other than 90° angle) if extended.

To further explore the above-mentioned spatial relationships between two planes, open the *Relations Between Two Planes.ipt* file (created in Autodesk Inventor program) inside the *Student Descriptive Geometry Learning Resource CD*, and use the **Rotate** tool to rotate to three-dimensional model (*Figure 1D-2B*).

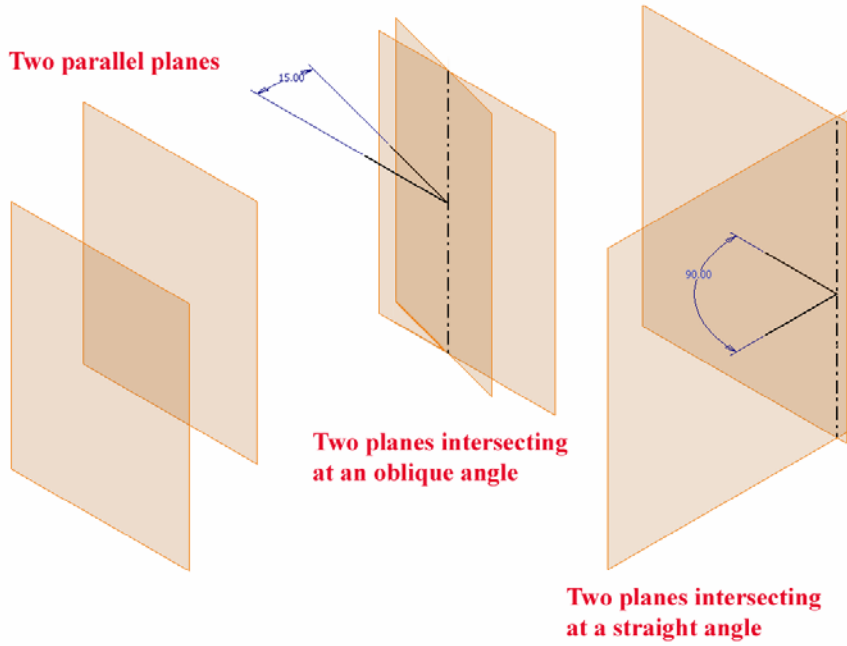


Figure 1D-2A: Relationships between two planes in three-dimensional space.

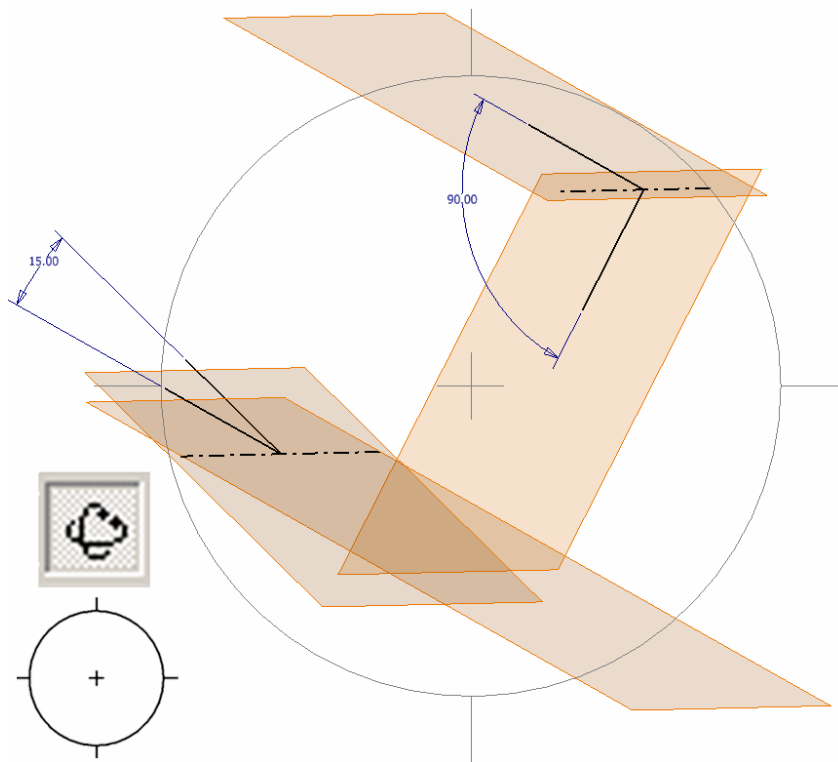


Figure 1D-2B: Using the Rotate tool to explore the spatial relations between two planes.

Definition of “dihedral angle” and basic strategy to determine in two-dimensional projection drawings the dihedral angle between two intersecting planes which exist in three-dimensional space

The word dihedral is composed of “di-” (meaning two) and “-hedral” (meaning “faces”). “Dihedral angle” refers to the true angle between two intersecting planes, measured from the cross-section that is perpendicular to the intersection (a straight linear edge shared by both planes). The key to finding the true angle between two planes is to draw several auxiliary views so as to show each plane as an edge view, and the intersection as a point end view.

In this Section, the Top and Front views of two planes A-B-C and B-C-D are given (*Figure 1D-2C*); and both are foreshortened. Therefore, we need to first create a primary auxiliary view, which is projected parallel to the intersection (edge line B-C) of the Front view, so as to obtain the true-length view of the intersection line B-C; next, we need to create a secondary auxiliary view that is projected perpendicular to the true-length view of intersection line B-C, so as to turn it into a point view. The true length of intersection edge B-C can be obtain also through either the revolution method or the TL DIAG (true-length diagram) method, (this helps as a double-check for accuracy after the Primary Auxiliary view is drawn).

Procedures for determining the dihedral angle between two given planes

To start, in the *Points, Lines, & Planes in the Three-Dimensional Space.dwg* file (inside the *Given Problems* folder, in the *Student Descriptive Geometry Learning Resource CD*); use the **Zoom** tool to zoom in the area under the tile of *Determining the Angle between Two Planes (The Dihedral Angle) with Intersection between Two Planes Given*; and keep the **ORTHO** and **OTRACK** buttons beneath the **Command Line** turned on, and the **Node** running **Object Snap** in the **Drafting Settings** window turned on. Notice that in the given views, projection lines have been created from the apparent points of intersection between the edges A-C and B-D across the Top and Front views, so as to determine the visibility of segments of each edge on either the Top or the Front view. For details on determining the visibility of line segments, review the *Section 14 - Determining Intersection and Visibility of Two Lines* (Pages 1B-63 to 1B-72) in *Module 1B - Points, Lines, & Planes in the Three-Dimensional Space*.

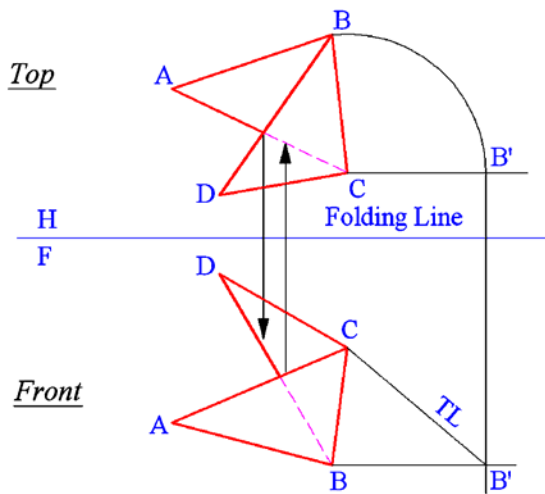


Figure 1D-2C: The given views of two planes.

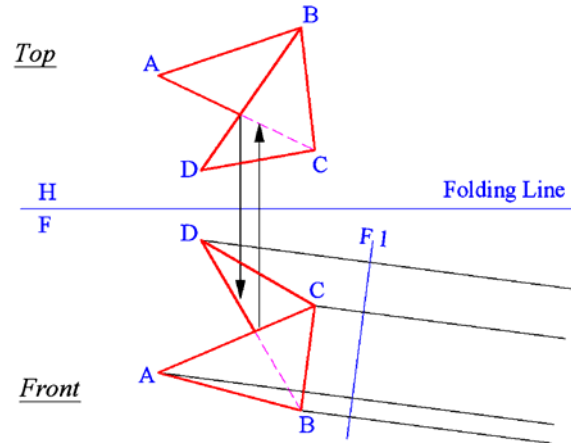


Figure 1D-2D: Starting the Primary Auxiliary view parallel to the foreshortened edge line B-C in the Front view by obtaining the first set of dimensions from the Front view.

The step-by-step process to solve for the dihedral angle graphically in the three-dimensional projection drawing is explained as follows:

1st Step: Creating the Primary Auxiliary view (the true-length view of line B-C, the edge of intersection between planes A-B-C and B-C-D):

We need to create the Primary Auxiliary view parallel to the foreshortened edge line B-C in the Front view to obtain its true-length view. Use the **UCS** tool with the **3-Point** option and **endpoint Object Snap** to align the **UCS** with the endpoints of line B-C in the Front view and turn **ORTHO** on. Next, use the **Line** tool to create projection lines from points A, B, C and D in the Front view into the Primary Auxiliary view, and to create the H/1 Folding Line (Primary Auxiliary View Ref Line) perpendicular to these projection lines as shown in *Figure 1D-2D*. The first set of dimensions has been projected from the Front view for the Primary Auxiliary view. Save the file. Save often throughout this Module.

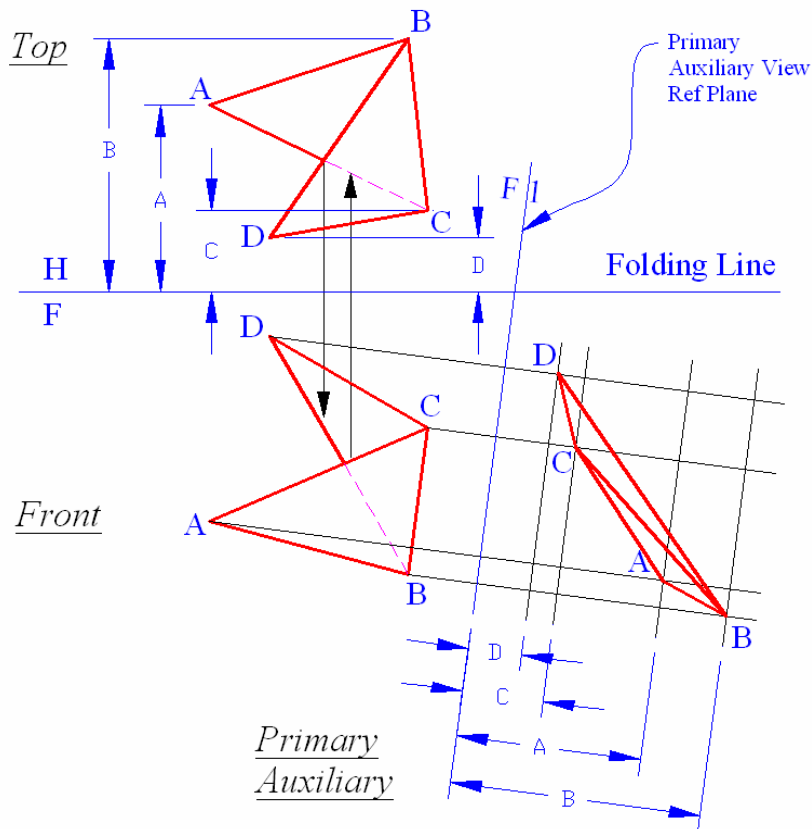
Next, obtain the second set of dimensions for the Primary Auxiliary view, from the Top view (the view adjacent to the Front view, from where the first set of dimensions has been projected). Return the **UCS** to the **World** settings; use the **Linear Dimension** tool to measure, in the Top view, the distances between the H/F Folding Line and points A, B, C and D respectively; then use the **Offset** tool to set off the same distances from the H/1 Folding Line (Primary Auxiliary View Ref Plane) as shown. Next, cross-referencing among the Top, Front and Primary Auxiliary views, determine the related points and then use the **Line** tool and appropriate **Object Snaps** to create the edges of the planes A-B-C

and B-C-D, in the Primary Auxiliary view. The intersection edge line B-C is now at true-length (abbreviated as TL, as shown in *Figure 1D-2E* and *Figure 1D-2F*).

2nd Step: Creating the Secondary Auxiliary view (the end point view of the intersection edge line B-C, and the edge views of both planes A-B-C and B-C-D):

We will now create the Secondary Auxiliary view, which is projected from the TL edge line B-C in the Primary Auxiliary view, to obtain the point end view of intersection edge line B-C.

To start, use the **UCS** tool with the **3-Point** option and **Endpoint Object Snap** to align the UCS with the endpoints of the TL line B-C in the Primary Auxiliary view and turn the **ORTHO** on. Next, use the **Line** tool to create projection lines from points A, B(C) and D into the Secondary Auxiliary view, and the folding line between the Primary and Secondary Auxiliary views, i.e., the 1/2 Folding Line (Secondary Auxiliary View Ref Plane). The first set of dimensions has been projected from the Primary Auxiliary view for the Secondary Auxiliary view. Save the file.



*Figure 1D-2E:
Obtaining the
second set of
dimensions
from the Top
view and
completing the
Primary
Auxiliary view.*

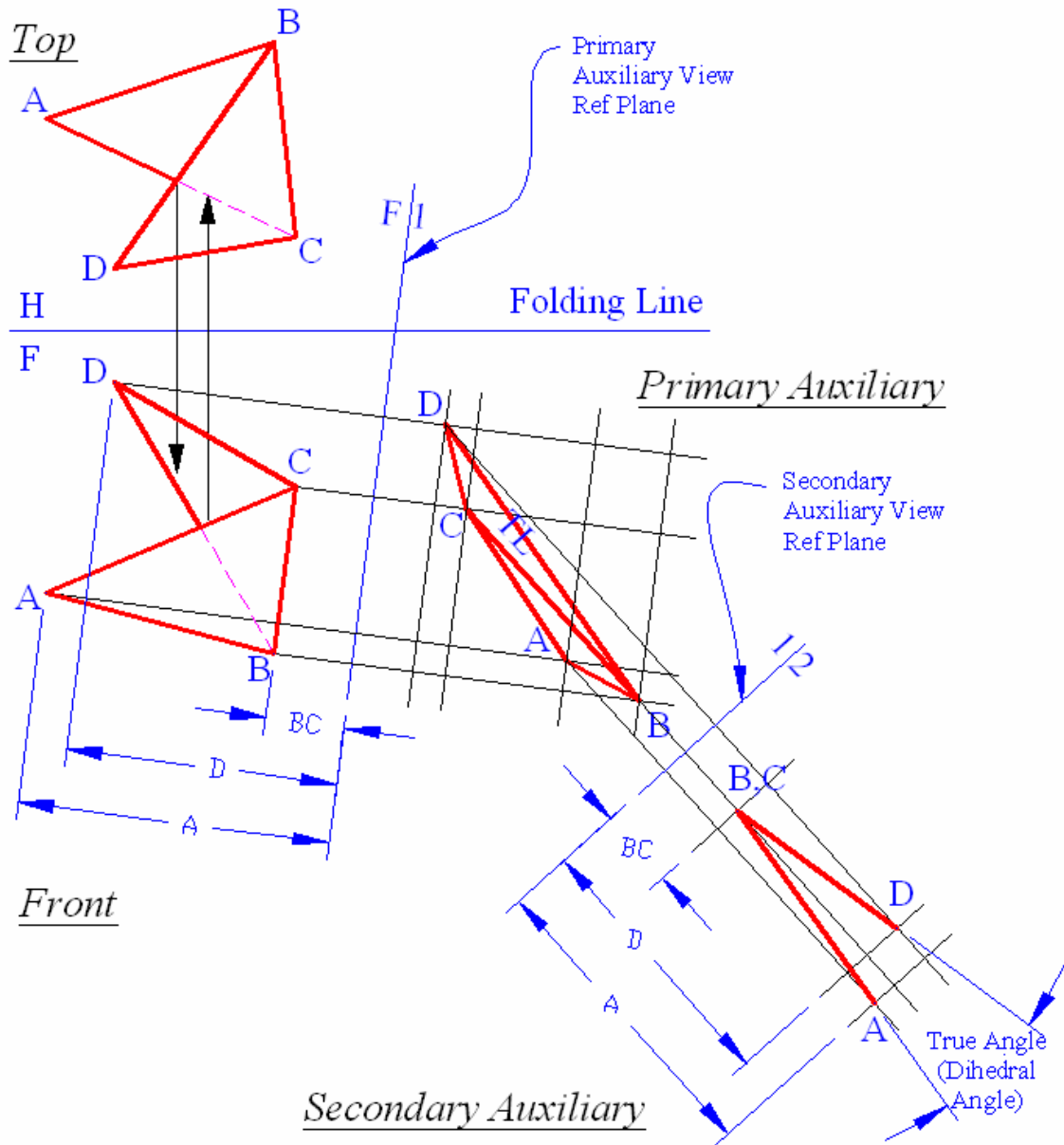


Figure 1D-2F: Completing the Secondary Auxiliary view (the end point view of of the line of intersection B-C, and the edge views of both planes A-B-C and B-C-D).

Next, obtain the second set of dimensions for the Secondary Auxiliary view, from the Front view (the view adjacent to the Primary Auxiliary view, from where the first set of dimensions has been projected). Use the **UCS** tool with the **3-Point** option and **Endpoint Object Snap** to align the UCS with the endpoints of the H/1 Folding Line

(Primary Auxiliary View Ref Plane); use the **Linear Dimension** tool to measure the distances from H/1 Folding Line to points A, B(C) and D respectively, in the Front view; next, use the **Offset** tool to set off the same distances from the 1/2 Folding Line (Secondary Auxiliary View Ref Plane) respectively, in the secondary Auxiliary view (*Figure 1D-2F*).

Next, cross-referencing among the Front, Primary and Secondary Auxiliary views, determine the related points in the Secondary Auxiliary view; and use the **Line** tool to create the plane edge lines A-B and B-D in the Secondary Auxiliary view. The intersection edge B-C between planes A-B-C and B-C-D is now in an end point view, and the planes A-B-C and B-C-D are at edge views.

Next, use the **Angular Dimension** tool to measure the true angle (dihedral angle) between both planes.

The Module is completed. Save and close the file.

This completes the Modules 1C. Congratulations! You have learned how to:

- Determine the angle between a line and a plane (given: the line and the plane in two principal views, both showing the line and the plane as foreshortened);
- Determine the angle between two planes (the dihedral angle) with intersection between two planes given.